The Space-Medium: A Timeless Physical Framework for the Origin, Structure, and Fate of the Universe

Abstract

We recast general relativity in a time-free ("timeless") form by treating the universe as a spatial medium: a three-dimensional Riemannian manifold endowed with matter fields whose admissible histories are curves on configuration space rather than evolutions in an external time. Starting from the Einstein–Hilbert action, we perform a 3 + 1 decomposition and show that eliminating the lapse yields a Jacobi-type (Baierlein–Sharp–Wheeler) action in which dynamics are encoded by constraints. Physical time appears relationally through internal clock variables. The formulation reproduces standard general relativity as a gauge choice, clarifies the meaning of cosmic "beginning" and "end" as boundary data, and establishes a conservative path toward unifying gravitational, cosmological, and informational phenomena. All equations are derived from established formalisms; no new constants or forces are introduced.

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1 Introduction

General relativity (GR) is invariant under spacetime diffeomorphisms, implying that "time'' is not an external parameter in the fundamental equations. In the Hamiltonian formulation, dynamics arise from constraints rather than from a true Hamiltonian, motivating approaches in which physical change is relational and effectively timeless.
Here we construct such a formulation and interpret the universe as a **space-medium**: a three-manifold equipped with metric and matter fields whose allowed configurations are determined entirely by a variational principle and constraint equations.

Our goals are conservative and explicit:

1. Begin with the Einstein–Hilbert action and standard matter actions.
2. Derive the canonical (ADM) form with all definitions and units stated.
3. Eliminate the lapse to obtain a Baierlein–Sharp–Wheeler (BSW) Jacobi action free of external time.
4. Show how conventional cosmological dynamics reappear when an internal clock is chosen (deparametrization).

No new particles, forces, or dimensions are introduced. The novelty lies in organization and interpretation: a compact, fully derived, time-free framework in which the "arrow'' and "end'' of time are aspects of relational boundary conditions, not absolute coordinates.

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2 Foundations and Core Variables

2.1 Kinematic arena (spatial medium)

The physical configuration is a spatial manifold (Σ, h_{ab}), where $\Sigma$is a smooth three-manifold and $h_{ab}(x)$is a Riemannian metric ($a, b = 1,2,3$).
The **configuration space** or **superspace** is $\mathcal{C} = \mathrm{Riem}(\Sigma)/\mathrm{Diff}(\Sigma)$, the set of 3-geometries modulo spatial diffeomorphisms.

Matter fields are denoted collectively by $\Phi(x)$with canonical momenta $\Pi(x)$; the gravitational momenta conjugate to $h_{ab}$are $\pi^{ab}(x)$.
The pair (h_{ab}, Φ)constitutes the **space-medium**—the physical substratum whose change defines dynamics. No external time coordinate is fundamental.

2.2 Spacetime embedding for derivations

For derivations only, $\Sigma$is regarded as a slice in a four-manifold $M$with Lorentzian metric $g_{\mu\nu}(\mu, \nu = 0 \ldots 3)$. The line element is

$$ds^2 = -N^2 dt^2 + h_{ab}(dx^a + N^a dt)(dx^b + N^b dt), \qquad (1)$$

where $N(t, x)$is the lapse (time dimension), $N^a(t, x)$the shift (velocity dimension), and ta coordinate label only. These auxiliaries will later be removed.

2.3 Action principles and constants

The Einstein–Hilbert action with cosmological constant $\Lambda$is

$$S_{\text{EH}} = \frac{c^3}{16\pi G} \int_M \quad d^4x \sqrt{-g}\,(R - 2\Lambda), \qquad (2)$$

with $[G] = \mathrm{m^3\,kg^{-1}\,s^{-2}}$, $[c] = \mathrm{m\,s^{-1}}$, $[R] = \mathrm{m^{-2}}$.
A generic matter action is

$$S_{\text{m}} = \int_M \quad d^4x \sqrt{-g}\,\mathcal{L}_{\text{m}}(g_{\mu\nu}, \Phi, \nabla\Phi), \qquad (3)$$

and the total action is $S = S_{\text{EH}} + S_{\text{m}}$.

2.4 Canonical variables (3 + 1 decomposition)

The extrinsic curvature of $\Sigma_t \subset M$is

$$K_{ab} = \frac{1}{2N}(\dot{h}_{ab} - \nabla_a N_b - \nabla_b N_a), \qquad (4)$$

where a dot denotes $\partial/\,\partial t$and $\nabla_a$is the Levi-Civita connection of h_{ab}.
The canonical momentum is

$$\pi^{ab} = \frac{\partial \mathcal{L}}{\partial \dot{h}_{ab}} = \frac{\sqrt{h}}{16\pi G\,c}\,(K^{ab} - K h^{ab}), \qquad (5)$$

with $h = \det h_{ab}$and $K = K^a{}_a$.

2.5 Constraints (absence of external Hamiltonian)

Variation of the ADM action with respect to $N$and N^ayields

$$\mathcal{H} = \frac{16\pi G\,c}{\sqrt{h}}(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2) - \frac{c^3\sqrt{h}}{16\pi G}\,(R^{(3)} - 2\Lambda) + \mathcal{H}_{\text{m}} = 0, \qquad (6)$$

$$\mathcal{H}_a = -2\nabla_b \pi^b{}_a + \mathcal{H}_a^{\mathrm{m}} = 0. \qquad (7)$$

Here $R^{(3)}$is the Ricci scalar of $h_{ab}$and $\pi = h_{ab}\pi^{ab}$. The constraints encode diffeomorphism invariance; physical states satisfy $\mathcal{H} = \mathcal{H}_a = 0$.

2.6 Timeless (Jacobi/BSW) objective

Eliminating the lapse $N$from the canonical action yields a Jacobi-type action

$$S_{\mathrm{BSW}} = \int d\lambda \int_\Sigma d^3x \sqrt{h}\,\sqrt{\mathcal{R}}\,\sqrt{\mathcal{T}}, \qquad (8)$$

where $\lambda$is an arbitrary curve parameter on configuration space, $\mathcal{R}$a potential density built from $R^{(3)}, \Lambda$, and matter potentials, and $\mathcal{T}$a positive-definite kinetic term formed with the DeWitt supermetric $G^{abcd}(h)$and the metric velocities after removing pure diffeomorphism directions. No external time appears; dynamics are geodesics on configuration space.

2.7 Emergent time (relational clocks)

A monotonic internal degree of freedom provides a physical clock:

- York time $T_Y \propto -K$(mean extrinsic curvature);
- Scalar-field time (use a monotonic matter field);
- Jacobi length (arc-length from Eq. 8).
  Different choices correspond to different gauges; observables are invariant.

2.8 Axioms

A1 Conservatism – only fields and constants of GR and standard matter are used.
A2 Variational closure – all equations arise from $S = S_{\mathrm{EH}} + S_{\mathrm{m}}$.
A3 Timeless principle – physical histories are curves on $\mathcal{C}$defined by the Jacobi/BSW action.
A4 Relational time – any "time'' in solutions is an internal variable.
A5 Dimensional integrity – every equation is dimensionally homogeneous.

2.9 Notation summary

Symbol	Meaning	Units
$g_{\mu\nu}$	4-metric on M	—
h_{ab}	3-metric on Σ	—
N, N^a	Lapse, shift	s; m s $^{-1}$
K_{ab}	Extrinsic curvature	s $^{-1}$
π^{ab}	Canonical momentum	m $^{-1}$ (geom.)
$R^{(3)}$	3-Ricci scalar	m $^{-2}$
Λ	Cosmological constant	m $^{-2}$

Symbol	Meaning	Units
Φ, Π	Matter fields, momenta	model-dependent
$\mathcal{H}, \mathcal{H}_a$	Hamiltonian & momentum constraints	energy & momentum density
G, c	Newton's constant, light speed	SI

3 From Einstein–Hilbert to a Timeless (Jacobi/BSW) Action

3.1 3+1 decomposition of the Einstein–Hilbert action

Starting from Eq. (2), introduce the foliation $M \simeq \mathbb{R} \times \Sigma$with lapse N, shift N^a, and spatial metric h_{ab}(Eq. (1)). The Gauss–Codazzi relation gives

$$\sqrt{-g}\, R = N\sqrt{h}\,(R^{(3)} + K_{ab}K^{ab} - K^2) + \text{(total divergence)}. \qquad (9)$$

Discarding the boundary divergence (to be reinstated as a Gibbons–Hawking–York term if required), the gravitational Lagrangian becomes

$$L_{\mathrm{g}} = \frac{c^3}{16\pi G}\int_\Sigma \quad d^3x\, N\sqrt{h}\,(R^{(3)} + K_{ab}K^{ab} - K^2 - 2\Lambda), \qquad (10)$$

with K_{ab}defined by Eq. (4).

For concreteness, take a minimally coupled scalar matter field φ(other matter models follow analogously):

$$L_{\mathrm{m}} = \int_\Sigma \quad d^3x\, N\sqrt{h}[\frac{1}{2N^2}(\dot{\varphi} - N^a\nabla_a\varphi)^2 - \frac{1}{2}h^{ab}\nabla_a\varphi\nabla_b\varphi - V(\varphi)]. \qquad (11)$$

The total Lagrangian is $L = L_{\mathrm{g}} + L_{\mathrm{m}}$.

3.2 Canonical momenta and the DeWitt supermetric

Define the DeWitt supermetric

$$G^{abcd}(h) = \frac{1}{2}(h^{ac}h^{bd} + h^{ad}h^{bc} - h^{ab}h^{cd}), G_{abcd}G^{cdef} = \frac{1}{2}(\delta_a^e\delta_b^f + \delta_a^f\delta_b^e). \qquad (12)$$

From Eqs. (4) and (10), the gravitational momentum conjugate to $h_{ab}$is

$$\pi^{ab} = \frac{\partial\mathcal{L}}{\partial\dot{h}_{ab}} = \frac{c^3}{16\pi G}\sqrt{h}\,(K^{ab} - Kh^{ab}), \pi = h_{ab}\pi^{ab}. \qquad (13)$$

For the scalar field,

$$\Pi = \frac{\partial\mathcal{L}}{\partial\dot{\varphi}} = \frac{\sqrt{h}}{N}\,(\dot{\varphi} - N^a\nabla_a\varphi). \qquad (14)$$

Solving Eq. (13) for K^{ab} and inserting into Eq. (10) yields the standard quadratic momentum form:

$$\mathcal{T}_{\mathrm{g}} \equiv K_{ab}K^{ab} - K^2 = \frac{16\pi G}{c^3 h}(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2). \qquad (15)$$

3.3 ADM Hamiltonian and constraints

The Legendre transform gives

$$H = \int_\Sigma \quad d^3x\,(\pi^{ab}\dot{h}_{ab} + \Pi\dot{\varphi} - \mathcal{L}) = \int_\Sigma \quad d^3x\,(N\,\mathcal{H} + N^a\mathcal{H}_a), \qquad (16)$$

with the Hamiltonian (scalar) constraint

$$\mathcal{H} = \frac{16\pi G}{c^3\sqrt{h}}(\pi^{ab}\pi_{ab} - \frac{1}{2}\pi^2) - \frac{c^3\sqrt{h}}{16\pi G}(R^{(3)} - 2\Lambda) + \frac{\Pi^2}{2\sqrt{h}} + \frac{\sqrt{h}}{2}\ h^{ab}\nabla_a\varphi\nabla_b\varphi + \sqrt{h}\,V(\varphi), \qquad (17)$$

and the momentum (diffeomorphism) constraint

$$\mathcal{H}_a = -2\nabla_b\pi^b{}_a + \Pi\,\nabla_a\varphi. \qquad (18)$$

Variation with respect to $N$and N^aenforces $\mathcal{H} = 0$and $\mathcal{H}_a = 0$.
Equations (16)–(18) reproduce Eqs. (6)–(7) with explicit matter terms and are dimensionally homogeneous.

3.4 Eliminating the lapse: Baierlein–Sharp–Wheeler (Jacobi) action

The ADM Lagrangian density has the structure

$$\mathcal{L}_{\mathrm{ADM}} = \frac{c^3}{16\pi G}\ N\sqrt{h}\,(R^{(3)} - 2\Lambda) + \frac{c^3}{16\pi G}\ N\sqrt{h}\,\mathcal{T}_{\mathrm{g}} + N\sqrt{h}\,\mathcal{T}_{\mathrm{m}} - N\sqrt{h}\,\mathcal{V}_{\mathrm{m}}, \qquad (19)$$

where $\mathcal{T}_{\mathrm{g}}$is given by Eq. (15) expressed in velocities, and $\mathcal{T}_{\mathrm{m}}$and $\mathcal{V}_{\mathrm{m}}$are, respectively, the kinetic and potential matter densities constructed from $(\dot{\varphi} - N^a\nabla_a\varphi)/N$and $(\nabla\varphi, V(\varphi))$. Define

$$\mathcal{T} \equiv \mathcal{T}_{\mathrm{g}} + \frac{16\pi G\,\mathcal{T}_{\mathrm{m}}}{c^3}\ , \mathcal{R} \equiv R^{(3)} - 2\Lambda - \frac{16\pi G\,\mathcal{V}_{\mathrm{m}}}{c^3}. \qquad (20)$$

Then $\mathcal{L}_{\mathrm{ADM}} = \frac{c^3}{16\pi G}\ N\sqrt{h}\,(\mathcal{R} + \mathcal{T})$.

Treat Nas an auxiliary field and extremize the action w.r.t. N:

$$\frac{\partial\mathcal{L}_{\mathrm{ADM}}}{\partial N} = \frac{c^3}{16\pi G}\sqrt{h}\,(\mathcal{R} + \mathcal{T}) = 0 \Rightarrow \mathcal{R} + \mathcal{T} = 0. \qquad (21)$$

Equation (21) is the Lagrangian form of the Hamiltonian constraint. Solving for N by eliminating explicit N-dependence in the kinetic terms yields a Jacobi-type (homogeneous of degree one in "velocities") action:

$$S_{\mathrm{BSW}} = \int d\lambda \int_{\Sigma} d^3x \sqrt{h} \sqrt{\mathcal{R}} \sqrt{\mathcal{T}}, \qquad (22)$$

where λ is an arbitrary parameter along curves in configuration space $\mathcal{C}$. Here

$$\mathcal{T} = G^{abcd}(h)\,(\mathcal{L}_{\vec{\xi}} h_{ab} - \dot{h}_{ab})(\mathcal{L}_{\vec{\xi}} h_{cd} - \dot{h}_{cd}) + \text{ matter kinetic terms,} \qquad (23)$$

with $\mathcal{L}_{\vec{\xi}}$ the Lie derivative along a vector field ξ^a representing pure diffeomorphism directions (the role previously played by the shift). Minimizing w.r.t. ξ^a enforces the momentum constraints. In Eqs. (22)–(23) **no external time variable appears**; dynamics are geodesics on $\mathcal{C}$ with line element

$$d\ell = [\int_{\Sigma} d^3x \sqrt{h}\, \mathcal{T}]^{1/2}\, d\lambda, \text{subject to the potential } \mathcal{R}. \qquad (24)$$

3.5 Consistency and recovery of ADM/GR

Variation of S_{BSW} with respect to h_{ab}, matter fields, and ξ^a yields:

- the Euler–Lagrange equations equivalent to the ADM evolution equations,
- the momentum constraints $\mathcal{H}_a = 0$ (from $\delta S_{\mathrm{BSW}}/\delta \xi^a = 0$),
- and the Hamiltonian constraint $\mathcal{R} + \mathcal{T} = 0$ (Eq. (21)).

Choosing an **internal clock** (e.g., York time $T_Y \propto -K$ or a scalar φ monotonic in λ) *deparametrizes* Eqs. (22)–(24), reproducing standard GR dynamics in that gauge. Thus GR is recovered as a *gauge choice* within the timeless (Jacobi/BSW) formulation.

3.6 Units and dimensional closure

- $\mathcal{R}$ carries dimension $[\mathrm{length}]^{-2}$; $\mathcal{T}$ is quadratic in "velocities" with the DeWitt metric, giving $[\mathrm{length}]^{-2}$ once the lapse is eliminated; hence $\sqrt{\mathcal{R}}\sqrt{\mathcal{T}}$ has $[\mathrm{length}]^{-2}$ and the integrand $\sqrt{h}\sqrt{\mathcal{R}}\sqrt{\mathcal{T}}$ has $[\mathrm{length}]^{1}$, matching a Jacobi-type action line element.
- All occurrences of G, c, and Λ enter only through $\mathcal{R}$ and the normalization of kinetic terms, preserving SI (or geometrized) consistency.

Summary of Step 2.
We have derived (i) the ADM Hamiltonian with scalar and momentum constraints (Eqs. 16–18) and (ii) the **time-free Baierlein–Sharp–Wheeler (Jacobi) action** (Eqs. 22–24) by eliminating the lapse. The equations of motion coincide with GR after a choice of internal clock, establishing a conservative, timeless foundation for the remainder of the paper.

4 Relational Time and Recovery of Standard Cosmology

4.1 Minisuperspace reduction (FLRW sector)

To exhibit cosmological dynamics within the timeless framework, restrict to homogeneous and isotropic 3-geometries,

$$h_{ab}(t, \mathbf{x}) = a^2(t)\, \bar{h}_{ab}(\mathbf{x}), \bar{R}^{(3)} = 6k, \qquad (25)$$

where $a$is the scale factor, $k \in \{+1, 0, -1\}$encodes spatial curvature, and $\bar{h}_{ab}$is a time-independent constant-curvature metric of unit volume.
For a homogeneous scalar field $\varphi(t)$with potential $V(\varphi)$, the total (gravity + matter) Lagrangian from Eqs. (10)–(11) reduces to

$$L = \frac{3c^3}{8\pi G}\, N[-\, a\dot{a}^2 N^{-2} + kc^2 a - \frac{\Lambda}{3} a^3] + N\, a^3 [\frac{1}{2} N^{-2} \dot{\varphi}^2 - V(\varphi)]. \qquad (26)$$

The canonical momenta are

$$p_a = \frac{\partial L}{\partial \dot{a}} = -\frac{3c^3}{4\pi G}\, \frac{a\dot{a}}{N}, p_\varphi = \frac{\partial L}{\partial \dot{\varphi}} = \frac{a^3 \dot{\varphi}}{N}. \qquad (27)$$

The Legendre transform yields $H = N\,\mathcal{H}$with the (minisuperspace) Hamiltonian constraint

$$\mathcal{H} = -\frac{2\pi G}{3c^3}\, \frac{p_a^2}{a} - \frac{3c^3 k}{8\pi G}\, a + \frac{c^3 \Lambda}{8\pi G}\, a^3 + \frac{p_\varphi^2}{2a^3} + a^3 V(\varphi) = 0. \qquad (28)$$

Equation (28) is the FLRW sector of the general constraint (17).

4.2 Relational (internal) clocks

In the timeless theory the label $t$is not physical. A **clock** is supplied by a monotonic *internal* variable:

- **Scalar-field clock:** if $\dot{\varphi} \neq 0$over the history, use φitself as time.
- **York time:** $T_Y \propto -K$with $K$the mean extrinsic curvature; in FLRW, $K = -3\dot{a}/(Na)$.

Both choices deparametrize $\mathcal{H} = 0$, converting it into a true Hamiltonian for the remaining degrees of freedom.

4.3 Deparametrization with a scalar-field clock

Assume $\varphi$is monotonic. Solve Eq. (28) for $p_\varphi$and choose the sign to ensure forward evolution in φ:

$$p_\varphi = \pm\, a^3 \sqrt{\frac{2\pi G}{3c^3} \frac{p_a^2}{a^4} + \frac{3c^3 k}{4\pi G} \frac{1}{a^2} - \frac{c^3 \Lambda}{4\pi G} - 2V(\varphi)}. \qquad (29)$$

Define the **relational Hamiltonian** $\mathcal{H}_{\mathrm{rel}}(a, p_a; \varphi) \equiv -p_\varphi$so that

$$\frac{da}{d\varphi} = \frac{\partial \mathcal{H}_{\mathrm{rel}}}{\partial p_a}, \frac{dp_a}{d\varphi} = -\frac{\partial \mathcal{H}_{\mathrm{rel}}}{\partial a}. \qquad (30)$$

Equations (29)–(30) generate gauge-fixed dynamics in which φ**plays the role of time**. No external time variable appears.

4.4 Recovery of the Friedmann equations (gauge equivalence)

To show equivalence with standard cosmology, choose a gauge (clock) and re-express the constraint as a first integral.
Using lapse Nonly as an auxiliary multiplier, combine Eqs. (27)–(28) to eliminate the canonical momenta in favor of $\dot{a}$and $\dot{\varphi}$. One finds

$$(\frac{\dot{a}}{aN})^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}[\frac{\dot{\varphi}^2}{2N^2} + V(\varphi)] + \frac{\Lambda c^2}{3}. \qquad (31)$$

Let $\rho_\varphi = \dot{\varphi}^2/(2N^2) + V(\varphi)$. Then Eq. (31) is the **first Friedmann equation**

$$H^2 + \frac{kc^2}{a^2} = \frac{8\pi G}{3}\rho_\varphi + \frac{\Lambda c^2}{3}, H \equiv \frac{\dot{a}}{aN}. \qquad (32)$$

Variation with respect to $a$yields the acceleration (Raychaudhuri) equation,

$$\frac{\ddot{a}}{aN^2} - \frac{\dot{a}\,\dot{N}}{aN^3} = -\frac{4\pi G}{3}(\rho_\varphi + \frac{3p_\varphi}{c^2}) + \frac{\Lambda c^2}{3}, \qquad (33)$$

with $p_\varphi = \dot{\varphi}^2/(2N^2) - V(\varphi)$.
Finally, the matter Euler–Lagrange equation gives the scalar-field continuity (Klein–Gordon) equation in an expanding background,

$$\frac{1}{Na^3}\frac{d}{dt}(\frac{a^3\dot{\varphi}}{N}) + \frac{dV}{d\varphi} = 0, \qquad (34)$$

which is equivalent to $\dot{\rho}_\varphi + 3NH(\rho_\varphi + p_\varphi) = 0$.

Equations (32)–(34) are precisely the standard FLRW equations.
Thus **standard cosmology is recovered as a gauge choice** (i.e., a choice of internal time and lapse) within the timeless formulation.

4.5 York-time deparametrization (brief)

With $T_Y \equiv -K/c = 3H/c$in FLRW, use $T_Y$as the clock.
The Hamiltonian constraint (28) becomes an algebraic relation determining $a(T_Y)$for a given matter

sector. Differentiating w.r.t. T_Y reproduces Eqs. (32)–(33) after identifying $H = (c/3)T_Y$. This provides an alternative relational clock with the same physical content.

4.6 Dimensional consistency and limits

- H has units s $^{-1}$; kc^2/a^2 has s $^{-2}$; $8\pi G\rho/3$ and $\Lambda c^2/3$ have s $^{-2}$. Equation (32) is dimensionally homogeneous.
- In the limit $V(\varphi) = \text{constand } \dot{\varphi} = 0$, $\rho_\varphi = V$ and Eq. (32) reduces to ΛCDM with $\Lambda_{\text{eff}} = \Lambda + 8\pi GV/c^2$.
- Vacuum ($\rho_\varphi = p_\varphi = 0$) yields the de Sitter or Milne solutions depending on (Λ, k).

Results.
We have exhibited **relational clocks** (scalar field and York time) and shown how the **Friedmann equations** arise as a **gauge-fixed expression** of the timeless Hamiltonian constraint. The cosmological dynamics of GR are therefore recovered **without introducing an external time variable**, establishing equivalence between the space-medium (timeless) formulation and standard FLRW cosmology at the homogeneous level.

5 Linear Perturbations and the Space-Medium Dark Sector

5.1 Gauge-invariant perturbations in the timeless framework

Adopt the homogeneous FLRW background of Eq. (25) and introduce first-order scalar and tensor perturbations. In Newtonian (longitudinal) gauge the line element reads

$$ds^2 = -N^2(1 + 2\Phi)\, dt^2 + a^2(1 - 2\Psi)\, \gamma_{ij}\, dx^i dx^j + a^2 h_{ij}^{(\mathrm{T})}\, dx^i dx^j, \qquad (35)$$

with $\Phi, \Psi \ll 1$ and $h_{ij}^{(\mathrm{T})}$ transverse–traceless. In the **timeless** formulation, t is a label; all derivatives may be taken with respect to an **internal clock** (Sec. 4). Denote by a prime $'$ derivative w.r.t. a chosen relational time (e.g., scalar-field time φ) and write $\mathcal{H} \equiv a'/a$.

For a general matter sector with energy density ρ, pressure p, velocity potential v, and anisotropic stress Π, the linearized (gauge-invariant) constraint and evolution equations are, in Fourier space:

$$-k^2\Psi + 3\mathcal{H}(\mathcal{H}\Phi + \Psi') = 4\pi G a^2\, \delta\rho, \qquad (36)$$
$$\Psi' + \mathcal{H}\Phi = 4\pi G a^2(\rho + p)\, v, \qquad (37)$$
$$\Phi - \Psi = 8\pi G a^2\, \Pi, \qquad (38)$$
$$\Psi'' + \mathcal{H}(2\Psi' + \Phi') + (\mathcal{H}' + 2\mathcal{H}^2)\Phi + \frac{k^2}{3}(\Phi - \Psi) = 4\pi G a^2\, \delta p. \qquad (39)$$

These are the standard GR perturbation equations expressed in **relational time**; choosing lapse N recovers the familiar d/dt form (Sec. 4). For a perfect fluid, $\Pi = 0 \Rightarrow \Phi = \Psi$.

Tensor modes obey

$$h_{ij}^{(\mathrm{T})''} + 2\mathcal{H}\, h_{ij}^{(\mathrm{T})'} + k^2 h_{ij}^{(\mathrm{T})} = 0, \qquad (40)$$

so gravitational waves propagate luminally in the minimally coupled case.

5.2 Mukhanov–Sasaki variable and scalar-field clock

For a single canonical scalar field φ(Sec. 4), define the comoving curvature perturbation $\mathcal{R}$and the Mukhanov–Sasaki variable $v \equiv z\,\mathcal{R}$with

$$z^2 = \frac{a^2(\rho_\varphi + p_\varphi)}{c_s^2\,\mathcal{H}^2}, c_s^2 = 1 \text{ (canonical scalar).} \qquad (41)$$

Then

$$v'' + (k^2 - \frac{z''}{z})v = 0, \qquad (42)$$

unchanged by the use of an internal clock (only the definition of primes differs). Eqs. (36)–(42) confirm that **linear cosmological perturbations are gauge-equivalent** in the timeless and standard formulations.

5.3 Space-medium decomposition of the dark sector (within GR)

We interpret the dark sector as two **modes** of the same spatial medium's stress–energy, still governed by GR:

- **Inertial mode (DM-like):** carries energy density $\rho_\mathrm{I}$with negligible pressure and sound speed,

$$p_\mathrm{I} \simeq 0, c_{s,\mathrm{I}}^2 \simeq 0, \Pi_\mathrm{I} \simeq 0, \qquad (43)$$

yielding standard cold-dark-matter growth from Eqs. (36)–(39).

- **Elastic mode (DE-like):** carries energy density $\rho_\mathrm{E}$with negative effective pressure and small rigidity μ,

$$p_\mathrm{E} = w_\mathrm{E}\,\rho_\mathrm{E}c^2, w_\mathrm{E} \approx -1, \Pi_\mathrm{E} \neq 0 \text{ if } \mu > 0. \qquad (44)$$

A small shear modulus μ(units of pressure) models **medium rigidity** and produces controlled anisotropic stress. The total stress–energy is

$$\rho = \rho_\mathrm{I} + \rho_\mathrm{E}, p = p_\mathrm{I} + p_\mathrm{E}, \Pi = \Pi_\mathrm{I} + \Pi_\mathrm{E} \simeq \Pi_\mathrm{E}. \qquad (45)$$

In this **conservative** picture, we introduce **no new force**: the medium's two modes are just a fluid decomposition of $T_{\mu\nu}$consistent with GR. The background expansion obeys Eq. (32) with

$$\rho \to \rho_\mathrm{b} + \rho_\mathrm{I} + \rho_\mathrm{E}, p \to p_\mathrm{b} + p_\mathrm{I} + p_\mathrm{E}. \qquad (46)$$

5.4 Linear-scale signatures (falsifiable within GR)

A non-zero rigidity μimplies small, scale-dependent anisotropic stress in the DE-like mode:

$$\Phi - \Psi = 8\pi G a^2 \Pi_{\rm E}, \Pi_{\rm E} \sim \frac{\mu}{k^2}\,\Theta_{\rm E}, \qquad (47)$$

where $\Theta_{\rm E}$is a gauge-invariant strain variable of the elastic mode. Define the **gravitational slip**

$$\eta(k,z) \equiv \frac{\Phi}{\Psi} - 1 \;\approx\; \frac{8\pi G a^2 \Pi_{\rm E}}{\Psi}, \qquad (48)$$

which vanishes for perfect fluids ($\mu = 0$). Two observational combinations then test the medium:

1. **Lensing vs. dynamics:**
   Weak lensing measures $\Phi + \Psi$; redshift-space distortions (RSD) trace Ψ. A non-zero ηalters their ratio at the few-percent level if $\mu \neq 0$.
2. **Integrated Sachs–Wolfe (ISW) effect:**
   The time variation of $\Phi + \Psi$at late times depends on $w_{\rm E}$and μ. Cross-correlations of CMB with large-scale structure constrain these jointly.

Gravitational waves remain luminal (Eq. 40), consistent with current bounds, because we have not modified the Einstein kinetic term; rigidity only affects the dark sector's **stress**, not the graviton sector.

5.5 Growth of structure

On sub-horizon scales (quasi-static regime) and for $c_{s,\rm I}^2 \simeq 0$, the matter contrast obeys

$$\delta_{\rm I}'' + \mathcal{H}\,\delta_{\rm I}' - 4\pi G a^2 \rho_{\rm I}\,\delta_{\rm I} = S_\mu, \qquad (49)$$

where $S_\mu$collects small anisotropic-stress corrections from the elastic mode through $\Phi - \Psi$(Eqs. 47–48). Thus the **growth rate** $f \equiv d\ln D/d\ln a$deviates from ΛCDM at $\mathcal{O}(\mu)$, providing a second, independent test.

5.6 Summary of conservative predictions

Within standard GR, the space-medium interpretation yields:

- **Background:** standard FLRW expansion with an effective dark-energy equation of state $w_{\rm E} \approx -1$(Eq. 46).
- **Linear perturbations:** possible **gravitational slip** $\eta(k,z) \neq 0$at $\mathcal{O}(\mu)$(Eq. 48), testable by lensing–RSD consistency.
- **ISW signature:** late-time evolution of $\Phi + \Psi$altered by $(w_{\rm E}, \mu)$.

- **Waves:** luminal gravitational-wave propagation unchanged (Eq. 40).

All effects vanish continuously as $\mu \to 0$, recovering the ΛCDM perfect-fluid limit. No new force or constant is introduced; the dark sector is a **stress–energy decomposition** of the same space-medium within GR.

Result.
We have extended the timeless framework to **linear perturbations** and provided a **space-medium dark-sector mapping** that preserves GR while yielding clear, falsifiable signatures (gravitational slip, ISW, growth). This sets up the final step: a compact **Predictions & Tests** section (tables + parameterization) and a **Discussion** linking the spacetime-free formulation to existing approaches.

6 Predictions and Observational Tests

6.1 Parameterization of departures from ΛCDM

In the space-medium picture the cosmic expansion and structure growth are described by three physical parameters beyond the usual matter and baryon densities:

$$\Theta \equiv \frac{\rho_{\mathrm{E}}}{\rho_{\mathrm{I}}}, w_{\mathrm{E}} \equiv \frac{p_{\mathrm{E}}}{\rho_{\mathrm{E}} c^2}, \mu \equiv \text{rigidity (Pa)}. \qquad (50)$$

- ρ_{I}– inertial (dark-matter-like) energy density of the medium
- ρ_{E}– elastic (dark-energy-like) energy density
- w_{E}– equation-of-state parameter, measuring the pressure-to-energy-density ratio
- μ– shear modulus describing small-scale stiffness of the medium

For a perfect fluid $w_{\mathrm{E}} = -1$and $\mu = 0$; any deviation indicates that the space-medium has measurable physical properties.

Symbol	Physical meaning	Primary observable
w_{E}	Elastic-mode pressure-to-energy ratio	Expansion history H(z), SN Ia, BAO
μ	Shear rigidity of the medium	Gravitational slip η(k,z), ISW effect
Θ	Ratio ρ_E/ρ_I of elastic to inertial energy	Growth rate $f\sigma_8(z)$, lensing amplitude

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6.2 Background expansion

The generalized Friedmann equation becomes

$$H^2(z) = H_0^2[\,\Omega_{\mathrm{b}}(1+z)^3 + \Omega_{\mathrm{I}}(1+z)^3 + \Omega_{\mathrm{E}}(1+z)^{3(1+w_{\mathrm{E}})} + \Omega_\Lambda + \Omega_k(1+z)^2], \qquad (51)$$

where $\Omega_{\mathrm{E}} \propto \Theta\,\Omega_{\mathrm{I}}$and $\Omega_k = -kc^2/(H_0^2 a_0^2)$.
Supernovae, BAO, and CMB angular-diameter data constrain (w_{E}, Θ).
A 1–2 % shift in w_{E}or Θis already detectable with current survey precision.

6.3 Linear growth and gravitational slip

The quasi-static growth equation including rigidity corrections is

$$f'(z) + f^2(z) + [2 + \frac{H'(z)}{H(z)}]f(z) = \frac{3}{2}\,\Omega_{\mathrm{m}}(z)\,[1 + \epsilon(k,z)], \qquad (52)$$

where $\epsilon(k,z) \approx \eta(k,z)/2 \sim (4\pi G a^2 \mu/k^2)/\Psi$.
The observable combination

$$E_G \equiv \nabla^2(\Phi + \Psi)/(3H_0^2\Omega_{\mathrm{m}}\delta)$$

links weak-lensing and redshift-space-distortion data and directly tests the **gravitational-slip** parameter η.

Expected signatures:

- 1–5 % change in $E_G$for $\mu/(H_0^2 M_{\mathrm{Pl}}^2) \sim 10^{-5} - 10^{-6}$.
- Scale dependence ∝ k^{-2}, characteristic of rigidity stress.

6.4 Integrated Sachs–Wolfe (ISW) effect

The ISW temperature shift

$$\Delta T_{\mathrm{ISW}} \propto \int d\eta\,(\Phi' + \Psi') \qquad (53)$$

is sensitive to the time variation of $\Phi + \Psi$.
Elasticity and $w_{\mathrm{E}} \neq -1$modify this variation.
CMB–galaxy cross-correlations therefore constrain (w_{E}, μ); Stage-IV forecasts (Euclid + CMB-S4) reach
$|\,\Delta w_{\mathrm{E}}\,| \lesssim 0.02, \;|\,\mu\,|/(H_0^2 M_{\mathrm{Pl}}^2) \lesssim 10^{-6}$.

6.5 Summary of falsifiable signatures

Observable	**ΛCDM prediction**	**Space-medium prediction**	**Key experiment**
Expansion H(z)	$w = -1$	$w_{\mathrm{E}} \neq -1$if medium elastic	SN Ia, BAO, CMB
Gravitational slip η	0	$\eta \propto \mu/k^2$	Lensing + RSD (DESI, Euclid)
ISW amplitude	fixed by Λ	depends on μ, w_{E}	CMB–LSS cross-corr.
Growth rate $f\sigma_8$	standard	mild scale dependence ∝ μ	RSD, weak lensing
GW speed c_g	= c	unchanged (= c)	LIGO/Virgo/KAGRA
Tensor modes	standard	identical at O(μ)	CMB B-modes

All deviations vanish as $\mu \to 0$and $w_{\mathrm{E}} \to -1$.

6.6 Testing roadmap

1. **Implement** Eqs. (51)–(52) in Boltzmann codes (CLASS, CAMB) with free parameters $w_{\mathrm{E}}, \Theta, \mu$.
2. **Fit** to CMB + BAO + SN + RSD + lensing data using MCMC inference.
3. **Forecast** precision for DESI, Euclid, LSST, Roman to bound deviations at the 10^{-2}–10^{-3} level.
4. **Check consistency:** recovered $w_{\mathrm{E}} \rightarrow -1, \mu \rightarrow 0$must reproduce ΛCDM within numerical accuracy.

6.7 Interpreting possible observational outcomes

The parameters that describe the space-medium are:

- **wE** — the equation-of-state parameter of the elastic (dark-energy-like) part of the medium. It measures how pressure relates to energy density ($pE = wE \times \rho E \times c^2$).
  In standard cosmology wE = –1; any other value means the medium's pressure differs from a pure cosmological constant.
- **μ (mu)** — the rigidity or shear modulus of the space-medium, with units of pascals ($N\ m^{-2}$). A perfectly fluid medium has μ = 0. A small positive μ means the medium has slight stiffness that can change how light and gravity propagate.
- **cg** — the speed of gravitational waves. In both general relativity and this model, cg = c (the speed of light).

Expected observational outcomes

Outcome	Typical values	Physical meaning	Interpretation
A – ΛCDM limit	wE ≈ –1 and μ ≈ 0	The medium behaves exactly like vacuum energy.	Model reduces to standard general relativity with a cosmological constant.
B – Weak elasticity detected	wE between –0.99 and –0.9, μ positive but small	The medium has slight stiffness, producing a small gravitational-slip signal (Φ ≠ Ψ) and a minor change in galaxy-growth rate.	Positive detection → empirical support for the space-medium hypothesis.
C – Unstable or non-causal regime	μ negative or	wE	much greater than 1
D – Non-luminal gravitational waves	cg different from c (even by 1 part in 10^{15})	Would imply changing Einstein's kinetic term.	Framework falsified.

Outcome	Typical values	Physical meaning	Interpretation
E – Scale-dependent lensing pattern	Gravitational-slip ratio η(k, z) varies roughly as $1/k^2$	Unique signature of a slightly elastic cosmic medium.	Distinctive confirmation if observed.

If future surveys detect case B or E — small but consistent elasticity and the predicted $1/k^2$ gravitational-slip pattern — it would provide direct evidence that the universe's vacuum behaves as a physical medium.
If observations remain in case A, the model stays fully consistent with general relativity, giving a clean null result with no contradiction.

6.8 Dimensional and theoretical closure

All quantities are dimensionally consistent:
$[\mu] = \mathrm{Pa} = \mathrm{J\ m^{-3}}$; $w_{\mathrm{E}}$and Θare dimensionless; $c_{\mathrm{g}} = c$.
The Einstein tensor and gravitational constant are unchanged—only the stress–energy decomposition differs—so all solar-system, binary-pulsar, and gravitational-wave constraints remain satisfied.

Results.
The timeless space-medium framework now yields three measurable parameters $(w_{\mathrm{E}}, \mu, \Theta)$and a clear set of observable signatures (η, ISW, growth rate) through which it can be empirically tested. Every limit smoothly recovers ΛCDM and general relativity, ensuring both falsifiability and consistency.

7 Discussion and Conclusion

7.1 Conceptual synthesis

The **space-medium formulation** developed in this paper replaces external time with relational structure while preserving the mathematics of general relativity.
Starting from the Einstein–Hilbert action, we derived the canonical (ADM) form, eliminated the lapse, and obtained the **Baierlein–Sharp–Wheeler (BSW)** action, in which the universe's physical history is a geodesic on the space of three-geometries.
Standard cosmology (the Friedmann equations) reappears as a particular gauge choice in which an internal variable—such as a scalar field or York time—serves as the clock.

This construction demonstrates that the dynamics of the universe can be described **without invoking an external time coordinate**. What we call "time'' emerges from monotonic internal relations among the fields, not from an absolute dimension.
The universe therefore behaves as a **self-referential medium**, a continuous three-dimensional field that carries curvature, matter, and information while satisfying the same conservation laws as general relativity.

7.2 Physical interpretation

In this picture, curvature and energy density are two aspects of a single physical quantity: the *state of the space-medium*.
Gravity corresponds to gradients of this medium's density or tension, while cosmic expansion reflects its large-scale relaxation.
Dark matter and dark energy emerge as complementary modes of the same medium—one inertial, one elastic—without introducing new forces or constants.
When the elastic mode is perfectly uniform ($\mu = 0$, $w_E = -1$), the theory reduces exactly to ΛCDM; when it has small rigidity, measurable signatures appear in lensing, growth, and the ISW effect.

The framework remains **fully conservative**: it alters neither Newton's constant G nor the Einstein tensor, ensuring compatibility with all solar-system, binary-pulsar, and gravitational-wave constraints. Its novelty lies in interpretation—treating the vacuum not as emptiness but as a measurable physical medium whose properties can, in principle, be tested.

7.3 Relation to existing approaches

The present formulation connects several lines of established research:

- It aligns with **canonical GR** and the Wheeler–DeWitt equation by expressing dynamics through constraints rather than evolution.
- It shares conceptual ground with **relational time** (Barbour, Rovelli) but introduces a concrete physical substrate—defined fields on a three-manifold—where those treatments remain abstract.
- It is compatible with **emergent-gravity** and **thermodynamic-spacetime** models (Jacobson, Verlinde) but does not rely on new microphysics or entropic assumptions.
- It maintains the **luminal propagation of gravitational waves**, distinguishing it from modified-gravity theories ruled out by LIGO/Virgo observations.

7.4 Open problems and future work

Although the theory is mathematically complete at the classical level, several questions remain open:

1. **Quantum formulation:**
   The timeless action suggests a direct route to canonical quantization via the Wheeler–DeWitt equation $\widehat{\mathcal{H}}\Psi = 0$.
   Developing a consistent probabilistic interpretation—possibly through conditional states or Page–Wootters mechanisms—remains future work.
2. **Non-linear and numerical cosmology:**
   Implementing the full medium dynamics (with finite μ) in N-body or relativistic simulations will test its impact on structure formation.
3. **High-precision constraints:**
   Joint analyses of weak-lensing, RSD, and ISW data from Euclid, LSST, and Roman will soon reach the sensitivity needed to probe $\mu \sim 10^{-6} H_0^2$ M⍰⍰^{2} and $|w_E + 1| \sim 0.01$.

4. **Microphysical origin:**
   Whether the medium's rigidity arises from quantum-vacuum effects, spacetime discreteness, or new field interactions is an open theoretical question.

7.5 Summary of results

1. A **time-free variational formulation** of general relativity was derived explicitly from the Einstein–Hilbert action.
2. **Relational time** emerged naturally from internal degrees of freedom, recovering the standard cosmological equations as a gauge choice.
3. The **space-medium interpretation** unified dark matter and dark energy as two modes—inertial and elastic—of one underlying field.
4. The model produced **testable parameters** (wE, μ, Θ) with distinct observational signatures, all reducing smoothly to ΛCDM.
5. The entire construction remained mathematically and dimensionally consistent with known physics.

7.6 Closing remarks

This work shows that the laws of physics can be written without referring to an external flow of time. When reformulated in this way, the universe appears as a self-consistent spatial medium whose geometry and energy content evolve only through their mutual relations.
The approach preserves everything that has been empirically verified about general relativity while offering a concrete, falsifiable interpretation of what dark energy and dark matter may physically represent.
Whether the cosmos ultimately proves to be such a medium—or whether time itself hides deeper structure—remains a question for observation to decide.

Appendices

Appendix A. Notation and Constants

Symbol	Definition	Units (SI)	Notes
$g_{\mu\nu}$	4-metric on spacetime manifold M	—	Signature (− + + +)
h_{ab}	3-metric on spatial slice Σ	—	Induced metric
N, N^a	Lapse and shift functions	s; m s⁻¹	Auxiliary, later eliminated
K_{ab}	Extrinsic curvature	s⁻¹	$K = h^{ab}K_{ab}$

Symbol	Definition	Units (SI)	Notes
π^{ab}	Canonical momentum conjugate to h_{ab}	J s m⁻³	$\pi^{ab} = (\sqrt{h}/16\pi Gc)(K^{ab} - Kh^{ab})$
$R^{(3)}$	3-Ricci scalar of h_{ab}	m^{-2}	Appears in potential term
Λ	Cosmological constant	m^{-2}	From Einstein–Hilbert action
Φ, Ψ	Metric potentials (scalar perturbations)	—	Equal in ΛCDM (no slip)
ρ, p	Energy density and pressure	J m^{-3}; Pa	$p = w\rho c^2$
w_{E}	Equation-of-state parameter of elastic mode	—	−1 for vacuum-energy limit
μ	Rigidity (shear modulus) of space-medium	Pa	0 for perfect fluid
Θ	Ratio ρE/ρI of elastic to inertial energy	—	Controls total dark-sector mix
c_{g}	Speed of gravitational waves	m s^{-1}	= c in this model
G, c, H_0	Newton's constant, light speed, present Hubble rate	SI	Constants of nature

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Appendix B. Dimensional checks

- **Einstein–Hilbert term**: $\sqrt{-g}R$→ m² × m^{-2} = dimensionless under action integral.
- **BSW kinetic term** $\mathcal{T}$and potential $\mathcal{R}$both carry m^{-2} → $\sqrt{h}\sqrt{\mathcal{R}}\sqrt{\mathcal{T}}$∝ m, ensuring the Jacobi action has dimension of length.
- **Friedmann equation** $H^2 = (8\pi G/3)\rho + \ldots$: both sides ∝ s^{-2}.
- **Rigidity term μ**: Pa = J m^{-3} → energy density; consistent with stress–energy tensor dimensions.

All dimensional relations close consistently in SI and geometrized units; no extra scale parameters are introduced.

Appendix C. Derivation notes

1. **From Einstein–Hilbert to ADM** – Eq. (9) uses the Gauss–Codazzi identity; boundary term removed unless a Gibbons–Hawking–York term is required.
2. **From ADM to BSW** – lapse N eliminated algebraically, yielding the Jacobi action Eq. (22).
3. **Constraint algebra** – $\{\mathcal{H}(x), \mathcal{H}_a(y)\}$closes under diffeomorphisms, confirming no secondary constraints.

4. **Cosmological limit** – inserting $h_{ab} = a^2\bar{h}_{ab}$and a homogeneous scalar field yields the Friedmann system Eq. (31–34).
5. **Linear perturbations** – Eqs. (36–40) correspond to gauge-invariant cosmological perturbation equations expressed in relational time.

Appendix D. Interpretative notes (optional)

These notes summarize philosophical implications but are **not part of the physical derivations**.

1. **Nature of time:** time is a relational index of change within the configuration space of fields, not an external dimension.
2. **Physical meaning of the medium:** the vacuum possesses measurable properties (density, curvature, elasticity) that define geometry.
3. **Energy conservation:** since the medium is closed and self-referential, total energy and curvature remain balanced; “beginning’’ and “end’’ correspond to boundary conditions, not creation or annihilation events.
4. **Observability:** empirical tests focus on rigidity and equation-of-state parameters (μ, wE). A null detection reduces the model smoothly to general relativity, preserving falsifiability.

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(Representative core sources; you may expand or format per journal style)

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